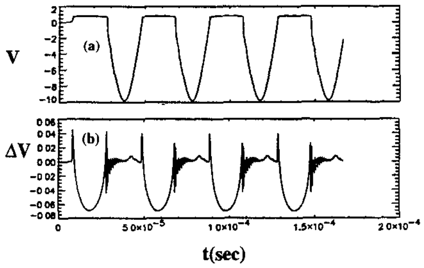

Fig. 2. (a) Time dependent voltage across a diode at the end of a transmission line. Solid: Ideal analytically integrated voltage. Dashed: PISCES integrated voltage, including external displacement current effects. (b) Deviation: $V_{PISCES} - V_{ideal} = \nabla V$.

the end of the line, and evaluated for 150 μsec. The solid line in Fig. 2(a) shows the resultant voltage across the diode. The signal is properly clipped at positive voltages when the diode is conducting.

Next, we used (5), and our abrupt-start trick to calculate $V^{(m+1)}$ and $E_y^{(m+1)}$ from PISCES. We plot the resultant diode voltage for the same sinusoidal input as the dashed curve in Fig. 2(a). Fig. 2(b) shows that the deviation from ideal is small, validating the new approach. The PISCES approach is, of course, much more general, and can be used with higher frequency input to calculate the net transient response of the coupled transmission line and device.

III. Conclusion

We have shown how a direct integration approach for simple lumped elements can improve the numerical properties of models employing FDTD analysis. More significantly, we have developed a noninvasive procedure by which standard PISCES-like software can be combined with FDTD to model the coupled internal dynamics of devices with external electromagnetics.

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On the Time Step in Hybrid Symmetrical Condensed TLM Nodes

Vladica Trenkic, Christos Christopoulos, and Trevor M. Benson

***Abstract*— New formulas for the maximum permissible time step in TLM hybrid nodes modeling anisotropic media are introduced and analyzed. It is shown that the value of the time step in most cases can be higher than that suggested by the minimum node dimension. The chosen value of the time step has significant impact on the dispersion characteristics of the hybrid symmetrical condensed node.**

I. Introduction

The hybrid symmetrical condensed node (HSCN) for the TLM method was originally described in [1]. Further generalizations of this node and a complementary HSCN were recently proposed in [2]. In the original HSCN [1], referred to as Type I in [2], all required inductances are modeled in the transmission lines, while open-circuit stubs are used to make up for any deficit in capacitances. A complementary Type II HSCN introduced in [2] models extra capacitances by altering the characteristic impedances of transmission-lines and uses short-circuit stubs to make up for any deficit in inductances.

The HSCN can operate with a larger time step than the stubbed SCN [3] due to the fact that the time step is not strictly dependent on the ratio of the largest to the smallest node dimension. Some considerations and comparisons of the maximum time step in the HSCN modeling isotropic media are given in [4], [5]. In the formulation of the HSCN for anisotropic media [2], the time step was related to the smallest mesh dimension Δl as $\Delta t = \Delta l/(2c)$. However, the maximum permissible time step was not defined.

In this paper we introduce the complete formulation for the maximum time step allowed in the HSCN for modeling anisotropic materials, based on the condition that characteristic admittances of the stubs must be nonnegative when modeling a passive medium [6]. We show that in most cases the value of the time step can be higher than that stated in [2]. Moreover, we demonstrate that dispersion characteristics of the HSCN are dependent on the chosen time step.

II. Maximum Time Step for HSCN

Contrary to the derivations in [1] and [2] where normalized characteristic admittances of stubs are given in terms of the smallest node dimension Δl, derivations in [5] are given directly in terms of the time step Δt. For an isotropic medium with electrical parameters ε_r, μ_r and a node with dimensions Δx, Δy, Δz, the normalized stub characteristic admittances for the Type I HSCN are given as [5]

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$$Yo_x = \frac{2\varepsilon_r \Delta y \Delta z}{c\Delta t \Delta x} - \frac{4c\Delta t\,[(\Delta y)^2 + (\Delta z)^2]}{\mu_r \Delta x \Delta y \Delta z} \quad (1)$$

$$Yo_y = \frac{2\varepsilon_r \Delta z \Delta x}{c\Delta t \Delta y} - \frac{4c\Delta t\,[(\Delta z)^2 + (\Delta x)^2]}{\mu_r \Delta x \Delta y \Delta z} \quad (2)$$

$$Yo_z = \frac{2\varepsilon_r \Delta x \Delta y}{c\Delta t \Delta z} - \frac{4c\Delta t\,[(\Delta x)^2 + (\Delta y)^2]}{\mu_r \Delta x \Delta y \Delta z} \quad (3)$$

where $c = 1/\sqrt{\mu_0 \varepsilon_0}$.

For an anisotropic medium described by diagonal tensors as in [2], the normalized characteristic admittances of the open-circuit stubs can be expressed in terms of the time step Δt as

$$Yo_x = \frac{2\varepsilon_{xx,r} \Delta y \Delta z}{c\Delta t \Delta x} - \frac{4c\Delta t}{\Delta x}\left(\frac{\Delta y}{\mu_{yy,r}\Delta z} + \frac{\Delta z}{\mu_{zz,r}\Delta y}\right) \quad (4)$$

$$Yo_y = \frac{2\varepsilon_{yy,r} \Delta z \Delta x}{c\Delta t \Delta y} - \frac{4c\Delta t}{\Delta y}\left(\frac{\Delta z}{\mu_{zz,r}\Delta x} + \frac{\Delta x}{\mu_{xx,r}\Delta z}\right) \quad (5)$$

$$Yo_z = \frac{2\varepsilon_{zz,r} \Delta x \Delta y}{c\Delta t \Delta z} - \frac{4c\Delta t}{\Delta z}\left(\frac{\Delta x}{\mu_{xx,r}\Delta y} + \frac{\Delta y}{\mu_{yy,r}\Delta x}\right). \quad (6)$$

If the time step Δt is related to the smallest node dimension Δl, as $\Delta t = \Delta l/(2c)$, then formulas (1)–(3) and (4)–(6) can be rewritten in the forms given in [1] and [2], respectively.

However, there is no particular reason for the time step to be related to the smallest node dimension. The only condition which must be met in formulating the characteristic admittances of the stubs when modeling a passive medium is that these admittances are nonnegative [6]. Hence the value of Δt must be chosen such that none of the characteristic admittance of the stubs becomes negative. Applying the conditions $Yo_x \geq 0$, $Yo_y \geq 0$, $Yo_z \geq 0$, it follows from (4)–(6) that

$$\Delta t \leq \frac{1}{2c}\sqrt{\frac{2\varepsilon_{xx,r}}{1/[\mu_{zz,r}(\Delta y)^2] + 1/[\mu_{yy,r}(\Delta z)^2]}} \quad (7)$$

$$\Delta t \leq \frac{1}{2c}\sqrt{\frac{2\varepsilon_{yy,r}}{1/[\mu_{xx,r}(\Delta z)^2] + 1/[\mu_{zz,r}(\Delta x)^2]}} \quad (8)$$

$$\Delta t \leq \frac{1}{2c}\sqrt{\frac{2\varepsilon_{zz,r}}{1/[\mu_{yy,r}(\Delta x)^2] + 1/[\mu_{xx,r}(\Delta y)^2]}}. \quad (9)$$

In general, to find the maximum time step for a TLM mesh, conditions (7)–(9), must be applied to all nodes throughout the mesh and the smallest time step found is the one that can be used.

The definitions of the normalized characteristic admittances in terms of Δl given in [1] and [2] can be readily used with an arbitrary time step Δt which satisfies relations (7)–(9). In this case, Δl, defined as $\Delta l = 2c\Delta t$, would be regarded as an *equivalent cubic cell parameter,* defined as the dimension of a cubic cell having the same propagation delay Δt as an arbitrarily graded TLM cell [7].

It can be easily confirmed that formulas (7)–(9) are also valid for the type II HSCN [2], provided one substitutes ε for μ and vice versa.

III. Analysis of the Expressions for the Maximum Time Step

We analyze expressions (7)–(9) in some detail for different mesh gradings and different media. For all cases, Δl is regarded as the smallest node dimension, while Δt_0, defined as $\Delta t_0 = \Delta l/2c$, is regarded as the time step related to the smallest node dimension. Let us recall that the time step for the uniform 3-D TLM mesh with node spacing Δl is given as $\Delta t = \Delta l/2c$ [3].

A. Homogeneous Isotropic Medium

We first analyze cases modeling a homogeneous isotropic medium with background properties $\varepsilon_{xx,r} = \varepsilon_{yy,r} = \varepsilon_{zz,r} = \mu_{xx,r} = \mu_{yy,r} = \mu_{zz,r} = 1$.

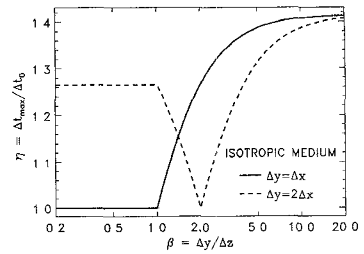

Fig. 1. Normalized maximum time step in an isotropic medium as a function of node aspect ratio.

Case h1: Two dimensions are the same and the third is bigger, e.g., $\Delta x = \Delta y = \Delta l$, $\Delta z > \Delta l$. The inequality (9) must be used, leading to

$$\Delta t_{\max} = \frac{\Delta l}{2c}. \quad (10)$$

Thus, for this case, the maximum time step $\Delta t_{\max}$ is the same as that related to the smallest node dimension, Δt_0.

Case h2: Two dimensions are the same and the third is smaller, e.g., $\Delta x = \Delta y = \Delta m > \Delta l$, $\Delta z = \Delta l$. The inequality (7) or (8) must be used, leading to

$$\Delta t_{\max} = \frac{1}{2c}\sqrt{\frac{2}{1/(\Delta m)^2 + 1/(\Delta l)^2}} > \frac{\Delta l}{2c}. \quad (11)$$

When $\Delta l \ll \Delta m$, the maximum time step becomes

$$\Delta t_{\max} \approx \frac{\Delta l\sqrt{2}}{2c}. \quad (12)$$

Case h3: All dimensions are different, $\Delta l = \Delta i < \Delta j < \Delta k$, where $i, j, k \in \{x, y, z\}$. If $(i, j, k) = (x, y, z)$, the inequality (9) must be used, leading to

$$\Delta t_{\max} = \frac{1}{2c}\sqrt{\frac{2}{1/(\Delta x)^2 + 1/(\Delta y)^2}} > \frac{\Delta l}{2c}. \quad (13)$$

From the Cases h1–h3 it can be seen that for any type of grading, the maximum permissible time step $\Delta t_{\max}$ lies within the limits

$$\frac{\Delta l}{2c} \leq \Delta t_{\max} < \frac{\Delta l\sqrt{2}}{2c} \quad (14)$$

which can also be expressed in terms of Δt_0

$$\Delta t_0 \leq \Delta t_{\max} < \Delta t_0\sqrt{2}. \quad (15)$$

This was also noted in [5]. Practically this means, that in some cases the time step can be chosen to be up to $\sqrt{2}$ times higher than one related to the smallest node dimension.

Cases h1–h3 are illustrated graphically in Fig. 1. The ratio η between the maximum permissible time step $\Delta t_{\max}$, as defined by formulas (7)–(9), and the time step Δt_0 related to the smallest node dimension is plotted against the aspect ratio β of the dimensions Δy and Δz. An increase in β can be interpreted as a decrease in Δz, whereas Δy and Δx remain constant. Note that β is presented on a logarithmic scale in Fig. 1.

Cases h1 and h2 ($\Delta y = \Delta x$) appear in Fig. 1 as the solid line. For $\beta < 1$, i.e., $\Delta z > \Delta y$, Case h1 applies and the ratio η remains unchanged and equal to unity. When $\beta > 1$, Case h2 applies and the time steps ratio increases up to $\eta = \sqrt{2}$. Case h3 is represented by the broken line in Fig. 1, for the example chosen $\Delta y = 2\Delta x$. Only at one point, when $\beta = 2$, i.e., $\Delta z = \Delta x$, does the maximum permissible time step, $\Delta t_{\max}$, equal Δt_0. Otherwise, when $\beta < 1$, formula (9) applies leading to $\eta = \sqrt{8/5} \approx 1.26$, whereas when $\beta \to \infty$, formula (8) applies leading to $\eta \to \sqrt{2}$.

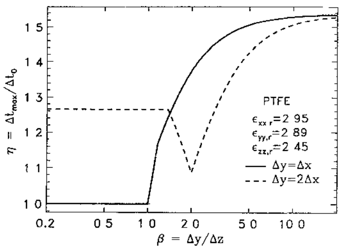

Fig. 2. Normalized maximum time step in anisotropic PTFE as a function of node aspect ratio.

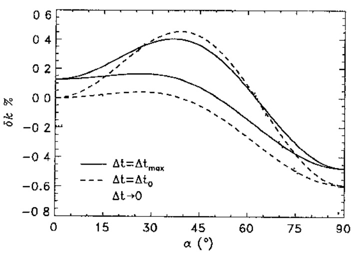

Fig. 3. Relative propagation error for different time steps and grading set at $\Delta x = 2\Delta y = \Delta z$.

B. Inhomogeneous Isotropic Media

When modeling inhomogeneous isotropic microwave circuits containing regions of different permeabilities and permittivities, $\varepsilon_{xx,r} = \varepsilon_{yy,r} = \varepsilon_{zz,r} = \varepsilon_r$ and $\mu_{xx,r} = \mu_{yy,r} = \mu_{zz,r} = \mu_r$, two cases can be considered.

Case i1: Grading ratio over different media is identical. In this case, it can be seen from formulas (7)–(9) that media with higher electrical and magnetic properties allow higher time steps, but the one related to the medium with the lowest permittivity and permeability (usually chosen as the background medium) must be used. Therefore, the analysis performed for the homogeneous Cases h1–h3 is valid for this inhomogeneous case too.

Case i2: Grading ratio over different media is different. In this case, full analysis of the permissible time steps for all node regions must be performed. It can be easily confirmed that the time step can always be chosen so as to be related to the smallest mesh dimension applied to the background medium parameters, but the actual configuration of the mesh might allow the use of a higher time step value.

C. Anisotropic Media

When modeling anisotropic materials, the analysis of the formulas (7)–(9) becomes more complex, so we only give an example for a material commonly used in microwave circuits, namely PTFE, with $\varepsilon_{xx,r} = 2.95$, $\varepsilon_{yy,r} = 2.89$, and $\varepsilon_{zz,r} = 2.45$ [8]. The time step Δt_0 is related to the smallest node dimension and the lowest electromagnetic parameters as

$$\Delta t_0 = \frac{\Delta l}{2c}\sqrt{\min_{i,j}\left(\varepsilon_{ii,r}\,\mu_{jj,r}\right)} \tag{16}$$

where $i, j \in \{x, y, z\}$. For the example of PTFE, plotted in Fig. 2, $\Delta t_0 = \Delta l\sqrt{\varepsilon_{zz,r}}/(2c)$. When $\beta \to \infty$, inequality (8) applies, leading to $\eta \to \sqrt{2\varepsilon_{yy,r}/\varepsilon_{zz,r}} \approx 1.536$.

IV. Impact on the Dispersion Characteristics

The relative propagation error for the HSCN is investigated for different values of time step by solving the general dispersion relation for the TLM SCN [9], with modifications to account for stubs [10].

An example corresponding to Case h2, similar to that in [11], with $\Delta x = 2\Delta y = \Delta z$ and a homogeneous medium ($\varepsilon_r = \mu_r = 1$), is considered for propagation along the coordinate plane $z = 0$. Fig. 3 shows the relative propagation error δk [11] in the HSCN operating on different time step values, calculated for the spatial discretization of $\Delta x/\lambda = 0.1$ and plotted as a function of the angle α between the propagation vector $\vec{k}$ and y-axis defined by $\alpha = \arctan(k_x/k_y)$. Note that two solutions of the general dispersion relation for the HSCN appear [10], [11]. Fig. 3 shows that the range of propagation error is the smallest when using the maximum time step $\Delta t_{\max}$.

V. Conclusion

It was shown that the value of the time step is not strictly dependent on the ratio of the smallest to the biggest node dimension as in the stubbed SCN, neither has it to be related to the smallest node dimension as described in [2]. New formulas for the allowable time steps were introduced for modeling general media using the HSCN and analyzed thoroughly for different problems. It was shown that in some isotropic cases the time step can be chosen to be up to $\sqrt{2}$ higher than that related to the smallest node dimension. In the case of anisotropic media this value can be even higher, depending on the material properties in the principal directions. It was shown that the dispersion characteristics of the HSCN are dependent on the time step value used in the mesh and that the maximum allowable time step for the HSCN yields the smallest range of propagation errors.

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